

# OCTONIONIC BINOCULAR MOBILEVISION. AN OVERVIEW.

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ABSTRACT. This paper is a compact overview of the heuristic approach to the recently elaborated octonionic binocular mobilevision [1].

## 1. INTERPRETATIONAL GEOMETRIES, ANOMALOUS VIRTUAL REALITIES, AND MOBILEVISION

### 1.1. Interpretational geometry.

A geometry being described below is related to a certain class of *interactive information systems*. Namely, let us call interactive information system *computer-graphic* if the information stream from the computer is mounted as a stream of geometric graphical data on a screen of the monitor; an interactive computer-graphic information system is called *psychoinformation* one if the information from observer to computer transmits unconsciously. Below we shall consider only such systems.

We shall define concepts of the interpretational figure and its symbolic drawings, which as it seems play a key role in the description of the computer-geometric representation of mathematical data in such interactive information systems.

Mathematical data in interactive information system exist in the form of an interrelation of *an interior geometric image (figure)* in the subjective space of observer and *an exterior computer-graphic representation*. The exterior computer-graphic representation includes *the visible elements* (drawings of figure) as well as of *the invisible ones* (f.e. analytic expressions and algorithms of the constructing of such drawings). Process of the corresponding of a geometrical image (figure) in the interior space of observer to a computer-graphic representation (visible and invisible elements) will be called *translation*. For example, a circle as a figure is a result of the translation of its drawing on a screen of the videocomputer (the visible object), constructed by an analytic formula (the invisible object) accordingly to the fixed algorithm (also the invisible object). It should be mentioned that the visible object may be nonidentical to the figure, f.e. if a 3-dimensional body is defined by an axonometry, in three projections, cross-sections or cuts, or in the window technique, which allows to scale up a concrete detail of a drawing, etc; in this case partial

visible elements may be regarded as *modules*, which translation is realized separately. We shall call the translation by *interpretation* if the translation of partial modules is realized depending on the result of the translation of preceeding ones and by *compilation* otherwise. An example of the interpretation may be produced by the drawing of a fractal which structure is defined by an observer on each step of the scaling up in the window technique; the translation of visible elements in an intentional anomalous virtual reality (see below) is also an interpretation.

**Definition 1.** A figure, which is obtained as a result of the interpretation, will be called *interpretational figure*.

It should be mentioned that an interpretational figure may have no any habitual formal definition; namely, only if the process of interpretation has an equivalent compilation process then the definition of figure is reduced to sum of definitions of its drawings; nevertheless, in interactive information systems not each interpretation process has an equivalent compilation one. It means that an interpretational figure has no any finite or constructively determined set of basic properties, from which other properties are derived in a formally logical way.

Note also that the drawing of interpretational figure may be characterized only as "visual perception technology" of figure but not as an "image", such drawings will be called *symbolic*.

The computer-geometric description of mathematical data in interactive information systems is deeply related to the concept of anomalous virtual reality.

It should be mentioned that there exist not less than two approaches to foundations of geometry: in the first one the basic geometric objects are figures defined by their drawings, geometry describes realtions between them, in the second one the basic geometric concept is a space (a medium, a field), geometry describes various properties of a space and its states, which are called the drawings of figures.

For the purposes of the describing of geometry of interactive information systems it is convenient to follow the second approach; the role of the medium is played by an anomalous virtual reality, the drawings of figures are its certain states.

## 1.2. Anomalous virtual realities.

### Definition 2.

A. *Anomalous virtual reality (AVR) in a narrow sense* is a certain system of rules of non-standard descriptive geometry adopted to a realization on videocomputer (or multisensor system of "virtual reality"). *Anomalous virtual reality in a wide sense* contains also an image in the cyberspace made accordingly to such system of rules. We shall use this term in a narrow sense below.

B. *Naturalization* is the corresponding of an anomalous virtual reality to an abstract geometry or a physical model. We shall say that the anomalous virtual reality *naturalizes* the model and such model *transcendizes* the naturalizing anomalous virtual reality. *Visualization in a narrow sense* is the corresponding of certain images or visual dynamics in the anomalous virtual reality to objects of the abstract geometry or processes in the physical model. *Visualisation in a wide sense* also includes the preceeding naturalization.

C. An anomalous virtual reality, whose images depends on an observer, is called *intentional anomalous virtual reality (IAVR)*. Generalized perspective laws in IAVR contain the equations of dynamics of observed images besides standard (geometric) perspective laws. A process of observation in IAVR contains a physical process of observation and a virtual process of intention, which directs an evolution of images accordingly to dynamical laws of perspective.

In the intentional anomalous virtual reality objects of observation present themselves being connected with observer, who acting on them in some way, determines, fixes their observed states, so an object is thought as a potentiality of a state from the defined spectrum, but its realization depends also on observer.

The symbolic drawings of interpretational figures are presented by states of a certain intentional anomalous virtual reality.

### 1.3. Colors in anomalous virtual realities.

It should be mentioned that the deep difference of descriptive geometry of computer-graphic information systems from the classical one is the presense of colors as important bearers of visual information. The reduction to shape graphics, which is adopted in standard descriptive geometry, is very inconvenient, since the use of colors is very familiar in the scientific visualization. The approach to the computer-graphic interactive information systems based on the concept of anomalous virtual reality allows to consider an investigation of structure of a color space as a rather pithy problem of descriptive geometry, because such space may be much larger than the usual one and its structure may be rather complicated. Also it should be mentioned that the using of other color spaces allows to transmit diverse information in different forms, so an investigation of the information transmission via anomalous virtual realities, which character deeply depends on a structure of color space, become also an important mathematical problem.

**Definition 3.** A set of continuously distributed visual characteristics of image in an anomalous virtual reality is called *anomalous color space*. Elements of an anomalous color space, which have non-color nature, are called *overcolors*, and quantities, which transcendize them in the abstract model, are called "*latent lights*". *Color-perspective system* is a fixed set of generalized perspective laws in fixed anomalous color space.

### 1.4. Mobilevision.

*Mobilevision* may be defined as a certain anomalous virtual reality, which naturalizes the so-called *quantum projective field theory* [1]. However, here we prefer to explicate such definition in more technical terms.

**Definition 4.** *Mobilevision* is an artificial computer-graphic interactive psychoinformation system with a projective invariant feedback determined by eye motions of observer.

Let's discuss this definition. First, mobilevision is an artificial interactive information system (this point corresponds to term "virtual" in the first form of the definition). Principles of its construction are self-consistent and do not copy automatically any natural laws just like *principles of airplane's construction differs*

from ones of bird's physiology (this point corresponds to the term "anomalous"). So mobilevision tries, first of all, to be a useful informatic construction but not a model of any (may be rather important) natural phenomena. Second, mobilevision is a computer-graphic information system, so an information stream from a computer to a human is mounted in a form of images on the screen; also it is a dynamical interactive system, i.e. the computer changes geometric data on the screen by a certain algorithm, and such changes depend on a behaviour of observer. Third, mobilevision is a psychoinformation interactive system, i.e. characteristics of human behaviour, which are available to the computer, have a subconscious character. Fourth, mobilevision is a very special psychoinformation system, a core of the subconscious information stream from a human to a computer is geometric, namely, consists of geometric data on eye motions. Such data may be reduced to the coordinates of a sight point on the screen and its velocity. Fifth, because both information streams in the mobilevision interactive system are essentially geometric, there is postulated a geometric correlation between them. Such correlation is encapsulated in dynamical laws of images realized by a certain algorithm. These law should be projectively invariant with respect to simultaneous projective transformations of image and sight geometric data. However, a self-evident claim of projective invariance does not specify the dynamical laws completely. Another invariance of dynamical laws is related to symmetries of a color space. At the first approximation one has (due to Maxwell, Helmholtz and Young) a  $SU(3)$  color symmetry (see par.2.1.), which is really, however, broken. Nevertheless, an approximate  $SU(3)$ -symmetry is a rather natural mathematical startpoint. Thus, one claims the dynamical laws of mobilevision to be  $SU(3)$ -invariant.

The described suppositions are sufficient for a mathematization of mobilevision, i.e. for a derivation of the using of all necessary mathematical requisites from the first principles of mobilevision.

First, let's represent all geometric continuously distributed data of image by certain quantities  $f_i(x, y)$ , where  $(x, y)$  are coordinates on the screen. It is convenient to use their chiral factorisation  $f_i(x, y) = \sum_{j,k} a_{ijk} \phi_j(z) \phi_k(\bar{z})$ , where  $\phi_j(z)$  are holomorphic functions of a complex variable  $z$ . The projective group  $PSL(2, \mathbb{C})$  (or, at least, its Lie algebra  $sl(2, \mathbb{C})$ ) acts on the quantities  $\phi_j(z)$  ("fields") as on holomorphic  $\lambda$ -differentials. The color group  $SU(3)$  also acts on them globally, i.e. transforms them by a rule independent on a point. Actions of  $sl(2, \mathbb{C})$  and  $SU(3)$  commute.

Second, let  $u$  be a complex coordinate of a sight point,  $\dot{u}$  be its velocity. It is rather natural to suppose that the dynamical laws are differential and that they express the first time-derivatives of "fields" as linear operators of "fields" themselves with coefficients depending on  $u$  and  $\dot{u}$ . The general form of such laws was written in [2]. The differential equations were interpreted as quantum-field analogs of the Euler formulas. It should be marked that a quantum-field meaning was given to these formulas by their interpretation and was not derived from general invariance principles. However, such interpretation is a useful source to pick out the most important cases of the dynamical laws. However, one may avoid it and to have deal with operators in dynamical laws in purely mathematical fashion as with the vertex operator fields for the Lie algebra  $sl(2, \mathbb{C})$ . Such vertex operator fields form

a certain algebraic structure (QPFT-operator algebra) described in details in [3]. However, the dynamical differential equations possess also  $SU(3)$  color symmetry, it manifests itself also as a symmetry of the related QPFT-operator algebra. QPFT-operator algebras with additional  $SU(3)$ -symmetries were described in [3] under the title of projective  $SU(3)$ -hypermultiplets. The most natural class of projective  $SU(3)$ -hypermultiplets (the canonical projective  $G$ -hypermultiplets,  $SU(3) \subset G$ ) was considered.

However, solitary Euler formulas are not  $SU(3)$ -invariant, so they should be completed by any other formulas. The most natural way to complete classical Euler formulas is to consider the Euler–Arnold equations. In our "quantum-field" case it means to consider the operators of dynamical laws (of the "quantum-field" Euler formulas) to be explicitly depending on a time, and to postulate their evolution to be governed by the Euler–Arnold equations [3]. The least have a hamiltonian form, and if a hamiltonian is  $SU(3)$ -invariant then the complete dynamical laws will be also  $SU(3)$ -invariant.

So the basic dynamical laws of mobilevision in a form of the "quantum-field" Euler formulas coupled with the Euler–Arnold equations are derived from the first principles. Note that "quantum-field" Euler formulas are fixed uniquely by the claim of projective invariance whereas the Euler–Arnold formulas may be replaced by any other ones, which will also provide the dynamical laws by  $SU(3)$ -invariance. Nevertheless, the Euler–Arnold formulas are, indeed, the most natural "anzatz".

Mark that mobilevision may be consider as a certain artificial form of *interactive visions*, which exploration is of a strong perpetual interest. F.e. it is rather intriguing to view this general topic in a context of multi-user effects in interpretational geometries.

Let's discuss mobilevision dynamics once more (cf.[2,4]).

First, note that the eye motions are not homogeneous. One may extract three different parts from them, namely, slow movements, saccads and tremor. The least may be naturally stochastized, i.e. be simulated by a certain stochastic process. It is resulted in an additional stochastic term in the Euler formulas. However, one may consider Euler formulas with an additional term from the beginning. In this case the dynamical laws are described by a stochastic linear differential equation of the form  $\dot{\Phi} = A(u, \dot{u})\Phi dt + B(u, \dot{u})\Phi d\omega$ , where operator fields  $A$  and  $B$  are independent (certainly, such equations are coupled with the deterministic Euler–Arnold equations on  $A$  to provide  $SU(3)$ -invariance). To maintain the  $SU(3)$ -invariance one should claim  $B$  to be  $SU(3)$ -invariant. Therefore, the most natural anzatz is to relate  $B$  to a  $SU(3)$ -invariant spin-1 vertex operator field (current) in the projective  $SU(3)$ -hypermultiplet. The resulted stochastic equations are formally a certain "quantum-field" analog of a form of Belavkin equations but without Belavkin counterterm, which provides exceptional nondemolition properties for solutions of Belavkin equations. Because this is the useful effect, we may include a "quantum-field" analog of Belavkin counterterm (determined by a spin-2  $SU(3)$ -invariant vertex operator field) in our equations by hands.

## 2. OCTONIONIC BINOCULAR MOBILEVISION

### 2.1. Quaternionic description of ordinary color space.

It should be mentioned that ordinary color space may be described by use of imaginary quaternions in the following way: let us consider an arbitrary imaginary complex quaternion  $q = ri + bj + gk$ ,  $i, j, k$  are imaginary roots and  $r, g, b$  are complex numbers. One may correspond to such quaternion an element of the color space, which in RGB-coordinates has components  $R = |r|^2$ ,  $G = |g|^2$ ,  $B = |b|^2$ . The lightening  $L$  has the quadratic form in the quaternionic space, namely,  $L = \frac{1}{2}(|r|^2 + |b|^2 + |g|^2)$ . The group  $SU(3)$  is a group of its invariance.

One may also consider any other coordinate systems XYZ in the color space such that RGB are linear combinations of XYZ. Then, the quaternion has the form  $q = xi + yj + zk$  and the lightening  $L$  is proportional to  $|x|^2 + |y|^2 + |z|^2$  (cf.[5]).

### 2.2. Octonionic color space and binocular mobilevision.

Let us construct an octonionic color space to describe the binocular mobilevision. This space will be a semi-direct product of a canonical projective  $G_2$ -hypermultiplet on the trivial one, which is a direct sum of seven copies of the suitable Verma module over  $\mathfrak{sl}(2, \mathbb{C})$ . The group  $G_2$  acts in this seven dimensional space as it acts on imaginary octonions [6]. There is uniquely defined up to a multiple and modulo the Poissonic center an  $SU(3)$ -invariant quadratic element in  $S^2(\mathfrak{g}_2)$ . So we can construct the Euler-Arnold equations in the canonical projective  $G_2$ -hypermultiplet. To receive the binocular version of the affine Euler formulas one should use the decomposition of  $S^2(\mathfrak{g}_2)$  on the  $SU(3)$ -chiral components (left and right); the angular fields from the chiral components will depend on chiral parameters  $u_l, \dot{u}_l$  and  $u_r, \dot{u}_r$ , attributed to the left and right eyes, respectively. Six copies of Verma modules over  $\mathfrak{sl}(2, \mathbb{C})$ , mentioned above, form a pair of projective  $SU(3)$ -hypermultiplets, which correspond to ordinary colors for left and right eyes; one copy form also a projective  $SU(3)$ -hypermultiplet, its overcolor will be called *a strange overcolor*. So the constructed seven dimensional octonionic color space includes a pair of ordinary three dimensional color spaces (for left and right eyes, respectively) and one strange overcolor.

Binocular mobilevision may be realised on any PC by use of two special components: (1) stereo glasses (f.e. 3Dmax of Kasan Electronics Co., Ltd.), and (2) any computer system of the real-time biomedical data acquisition. One may use dynamical perspective laws different from described above. Really, it is very convenient to use methods of dynamical interactive screening (analogous to noninteractive dynamical screening of 3Dmax).

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